LESSON 15.1

# Determining When Three Lengths Form a Triangle

**TEKS Expressions, equations, and relationships—6.8.A** Extend previous knowledge of triangles and their properties to include . . . determining when three lengths form a triangle.

**ESSENTIAL QUESTION**

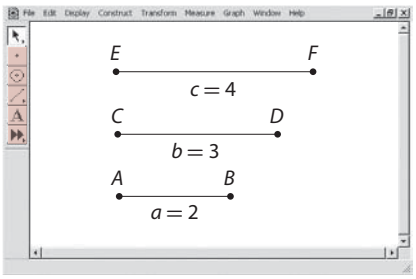
How can you use the relationship between side lengths to determine when three lengths form a triangle?

**EXPLORE ACTIVITY** TEKS 6.8.A

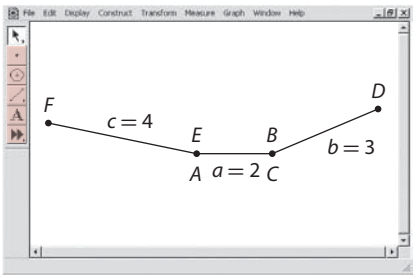
## Drawing Three Sides

**Use geometry software to draw a triangle whose sides have the following lengths: 2 units, 3 units, and 4 units.**

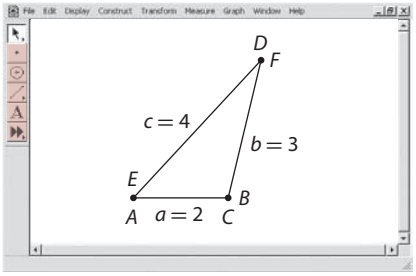
**A** Draw three line segments of 2, 3, and 4 units of length.




**B** Let $\overline{AB}$ be the base of the triangle. Place endpoint *C* on top of endpoint *B* and endpoint *E* on top of endpoint *A*. These will become two of the vertices of the triangle.




**C** Using the endpoints *C* and *E* as fixed vertices, rotate endpoints *F* and *D* to see if they will meet in a single point.




The line segments of 2, 3, and 4 units **do / do not** form a triangle.

**D** Repeat Steps 2 and 3, but start with a different base length. Do the line segments make the exact same triangle as the original?

The line segments **do / do not** make the same triangle as the original.

**E** Draw three line segments of 2, 3, and 6 units. Can you form a triangle with the given segments?

The line segments of 2, 3, and 6 units **do / do not** form a triangle.

**EXPLORE ACTIVITY** *(cont'd)*

### Reflect

1. **Conjecture** Try to make triangles using real world objects such as three straws of different lengths. Find three side lengths that form a triangle and three side lengths that do not form a triangle. What do you notice about the lengths that do not form a triangle?

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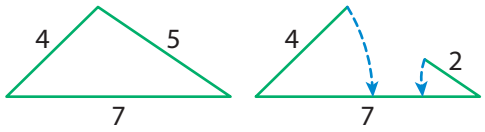
# Using Triangle Side Length Relationships

You saw in the Explore Activity that you cannot always form a triangle from three given line segments.

## Triangle Inequality

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.




Can form a triangle

Cannot form a triangle

You can use this relationship to determine if given side lengths can form a triangle.

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## EXAMPLE 1

TEKS 6.8.A

**Tell whether a triangle can have sides with the given lengths.**

**A** 11 cm, 6 cm, 13 cm

**STEP 1** Find the sum of the lengths of each pair of sides.

$11 + 6 \overset{?}{>} 13 \quad 6 + 13 \overset{?}{>} 11 \quad 11 + 13 \overset{?}{>} 6$

**STEP 2** Compare the sum to the third side.

$17 > 13$ ✓ $\quad 19 > 11$ ✓ $\quad 24 > 6$ ✓

The sum of any two of the given lengths is greater than the third length.

So, a triangle **can** have these side lengths.

**B** 5 ft, 15 ft, 9 ft

**STEP 1** Find the sum of the lengths of each pair of sides.

$5 + 15 \stackrel{?}{>} 9$ $\quad 15 + 9 \stackrel{?}{>} 5$ $\quad 5 + 9 \stackrel{?}{>} 15$

**STEP 2** Compare the sum to the third side.

$20 > 9$ ✓ $\quad 24 > 5$ ✓ $\quad 14 \not> 15$

The sum of any two of the given lengths is **not** greater than the third length.

So, a triangle **cannot** have these side lengths.

**Math Talk**
**Mathematical Processes**
Explain why a triangle with sides measuring 5 in., 5 in., and 1 foot cannot be constructed.

**YOUR TURN**

**Tell whether a triangle can have sides with the given lengths. Explain.**

**2.** 3 cm, 6 cm, 9 cm

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**3.** 4 m, 5 m, 8 m

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# Using Inequalities to Represent the Relationship Between Triangle Side Lengths

You can use what you know about the relationship among the lengths of the sides of a triangle to write an inequality. Then you can use the inequality to determine if a given value can be the length of an unknown side.

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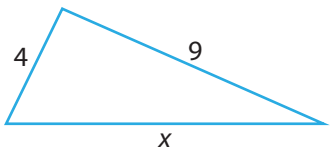
## EXAMPLE 2

TEKS 6.8.A

Which value could be the length of $x$?

$x = 15$ $\quad x = 10$



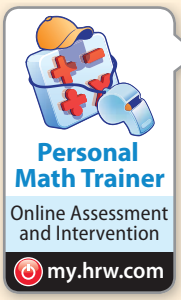

**STEP 1** $4 + 9 > x$ $\quad 4 + 9 > x$ *Write an inequality.*

**STEP 2** $4 + 9 \stackrel{?}{>} 15$ $\quad 4 + 9 \stackrel{?}{>} 10$ *Substitute each value for x.*

**STEP 3** $13 \not> 15$ $\quad 13 > 10$ ✓ *Compare the sum to the given value of x.*

The value that could be the length of $x$ is $x = 10$.

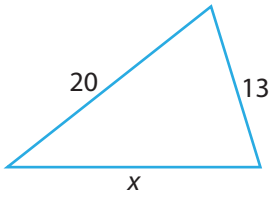
**Math Talk**
**Mathematical Processes**
Explain how you know that the Triangle Inequality relationship is true for every equilateral triangle.




## YOUR TURN

**4.** Which value could be the length of $x$?

$x = 35$ $x = 13$




## Guided Practice

**Determine whether a triangle can have sides with the given lengths. Explain.** (Explore Activity and Example 1)

**1.** 3 cm, 10 cm, 8 cm

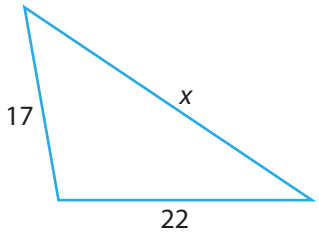
**2.** 10 ft, 10 ft, 18 ft

**3.** 30 in., 20 in., 40 in.

**4.** 16 cm, 12 cm, 3 cm

**5.** Which value could be the length of $x$? (Example 2)

$x = 29$ $x = 45$



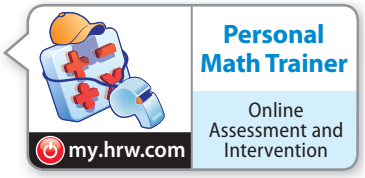

## ESSENTIAL QUESTION CHECK-IN

**6.** Explain how you can determine whether three metal rods can be joined to form a triangle.

Name _____ Class _____ Date _____

# 15.1 Independent Practice

TEKS 6.8.A




**7.** A map of a new dog park shows that it is triangular and that the sides measure 18.5 m, 36.9 m, and 16.9 m. Are the dimensions correct? Explain your reasoning.

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**8.** Choose a real world object that you can cut into three different lengths to form a triangle. Find three side lengths that form a triangle and three lengths that do not form a triangle. For each triangle, give the side lengths and explain why those lengths do or do not form a triangle.

Triangle 1: _____

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Triangle 2: _____

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**9.** Could the three sides of a triangular shopping mall measure $\frac{1}{2}$ mi, $\frac{1}{3}$ mi, and $\frac{1}{4}$ mi? Show how you found your answer.

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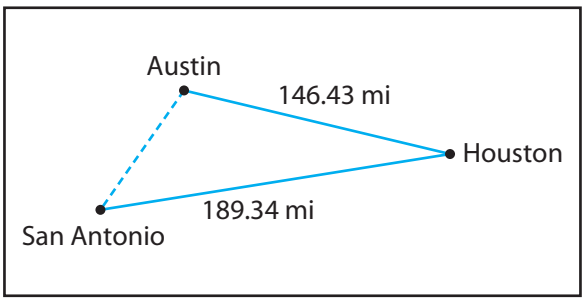
**10.** **Geography** The map shows the distance in air miles from Houston to both Austin and San Antonio.




**a.** What is the greatest possible distance from Austin to San Antonio?

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**b.** How did you find the answer?

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**c.** What is the least possible distance from Austin to San Antonio?

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**d.** How did you find the answer?

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**11.** **Critical Thinking** Two sides of an isosceles triangle measure 3 inches and 13 inches respectively. Find the length of the third side. Explain your reasoning.

## H.O.T. FOCUS ON HIGHER ORDER THINKING

**12.** **Critique Reasoning** While on a car trip with her family, Erin saw a sign that read, "Amarillo 100 miles, Lubbock 80 miles." She concluded that the distance from Amarillo to Lubbock is $100 - 80 = 20$ miles. Was she right? Explain.

**13.** **Make a Conjecture** Is there a value of $n$ for which there could be a triangle with sides of length $n$, $2n$, and $3n$? Explain.

**14.** **Persevere in Problem Solving** A metalworker cut an 8-foot length of pipe into three pieces and welded them to form a triangle. Each of the 3 sections measured a whole number of feet in length. How long was each section? Explain your reasoning.

Work Area