LESSON 15.3

Relationships Between Sides and Angles in a Triangle

TEKS **Expressions, equations, and relationships—6.8.A** Extend previous knowledge of triangles and their properties to include. . . the relationship between the lengths of sides and measures of angles in a triangle. . .

ESSENTIAL QUESTION

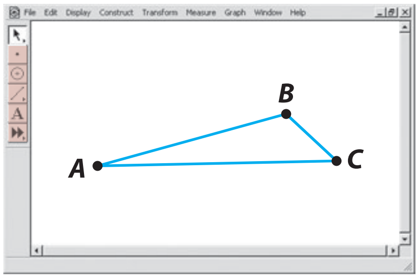
How can you use the relationships between side lengths and angle measures in a triangle to solve problems?

EXPLORE ACTIVITY TEKS 6.8.A

Exploring the Relationship Between Sides and Angles in a Triangle

There is a special relationship between the lengths of sides and the measures of angles in a triangle.

A Use geometry software to make triangle *ABC*. Make $\angle A$ the smallest angle.



B Choose one vertex and drag it so that you lengthen the side of the triangle opposite angle *A*. Describe what happens to $\angle A$.

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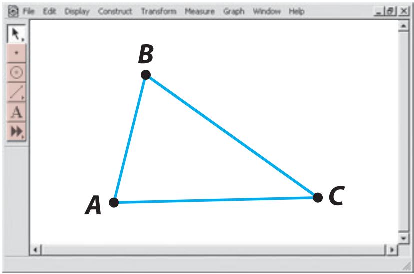

C Drag the vertex to shorten the side opposite $\angle B$. What happens to $\angle B$?

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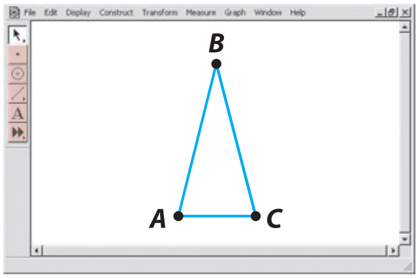

D Make several new triangles. In each case, note the locations of the longest and shortest sides in relation to the largest and smallest angles. Describe your results.

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My Notes

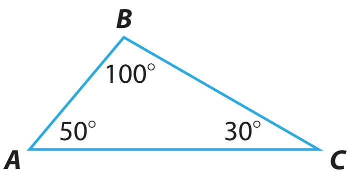
Using the Relationship Between Sides and Angles in a Triangle

You have seen that in a triangle the largest angle is opposite the longest side and the smallest angle is opposite the shortest side. It follows that the midsize angle is opposite the midsize side.

EXAMPLE 1

TEKS 6.8.A

A Triangle *ABC* has side lengths of 7 cm, 9 cm, and 4.5 cm. Use the relationship between the sides and angles of a triangle to match each side with its correct length.

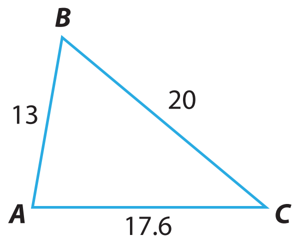


$AC = 9$ cm — The longest side is opposite the largest angle.

$AB = 4.5$ cm — The shortest side is opposite the smallest angle.

$BC = 7$ cm — The midsize side is opposite the midsize angle.

B Triangle *ABC* has angles measuring 60°, 80°, and 40°. Use the relationship between the sides and angles of a triangle to match each angle with its correct measure.



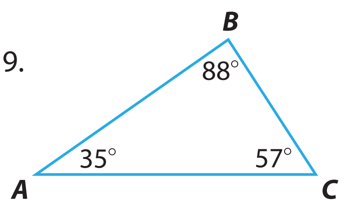
$m\angle A = 80°$ — The largest angle is opposite the longest side.

$m\angle C = 40°$ — The smallest angle is opposite the shortest side.

$m\angle B = 60°$ — The midsize angle is opposite the midsize side.

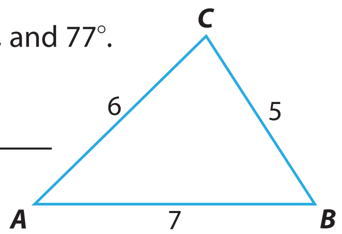
YOUR TURN

1. Triangle *ABC* has side lengths of 11, 16, and 19. Match each side with its correct length.



$AB =$ ______ $AC =$ ______ $BC =$ ______

2. Triangle *ABC* has angle measures of 45°, 58°, and 77°. Match each angle with its correct measure.



$m\angle A =$ ______ $m\angle B =$ ______ $m\angle C =$ ______

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Solving Problems Using Triangle Relationships

Recall that triangles can be classified by the lengths of their sides. A *scalene* triangle has no congruent sides. An *isosceles* triangle has two congruent sides. An *equilateral* triangle has three congruent sides.

EXAMPLE 2 Problem Solving

TEKS 6.8.A

Brandy is making a quilt. Each block of the quilt is made up of four triangles. Each triangle is in the shape of a right isosceles triangle. Two of the side measures of one triangle are 6.4 inches and 9 inches. Brandy wants to add a ribbon border around one of the triangles. How much ribbon will she need?

Analyze Information

Rewrite the question as a statement.

- Find the amount of ribbon Brandy will need for a border around one triangle.

Identify the important information.

- Each quilt piece has the shape of a right isosceles triangle.
- Two sides of the triangle measure 6.4 inches and 9 inches.

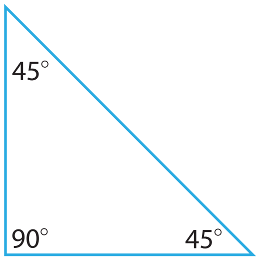
Formulate a Plan

You can draw a model and label it with the important information to find the total length of ribbon that Brandy needs for one triangle.

Solve

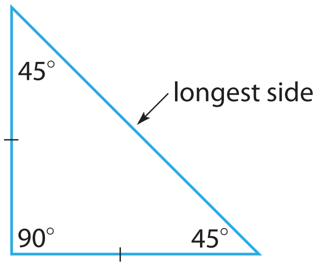
Think: A right triangle will have one 90° angle. Since the sum of the angles is 180°, the other two angles will be congruent and will have a combined measure of 90°.

$90° \div 2 = 45°$



Label the new information on the model.

90° is the greatest angle measure, so the side opposite the 90° angle will be the longest side. The other two angles are congruent, so the sides opposite those angles are congruent.



The shortest side lengths are 6.4 inches and 6.4 inches. So, Brandy will need $6.4 + 6.4 + 9 = 21.8$ inches of ribbon.

Justify and Evaluate

The solution is reasonable because the quilt piece is in the shape of an isosceles right triangle and it has two sides measuring 6.4 inches and 9 inches.

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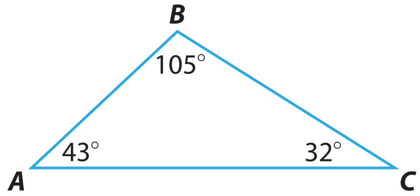
YOUR TURN

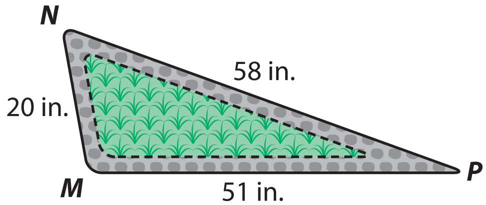
3. A fence around a rock garden is in the shape of a right triangle. Two angles measure 30° and 60°. Two sides measure 10 feet and 17.3 feet. The total length of the fence is 47.3 feet. How long is the side opposite the right angle? ______________________

Guided Practice



1. Triangle *ABC* has side lengths of 17, 13, and 24. Match each side with its correct length. (Example 1)

______ = 24 ______ = 13 ______ = 17



2. The figure represents a traffic island that has angles measuring 60°, 20°, and 100°. Match each angle with its correct measure. (Example 1)

m∠______ = 100° m∠______ = 20° m∠______ = 60°

3. **Vocabulary** Explain how the relationship between the sides and angles of a triangle applies to equilateral triangles. (Example 2)

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4. Ramone is building a fence around a vegetable garden in his backyard. The fence will be in the shape of a right isosceles triangle. Two of the side measures are 12 feet and 16 feet. Use a problem solving model to find the total length of fencing he needs. Explain. (Example 2)

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ESSENTIAL QUESTION CHECK-IN

5. Describe the relationship between the lengths of the sides and the measures of the angles in a triangle.

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15.3 Independent Practice

TEKS 6.8.A

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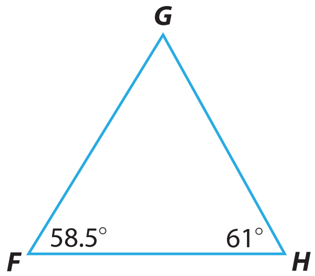
Use the figure for 6–8.

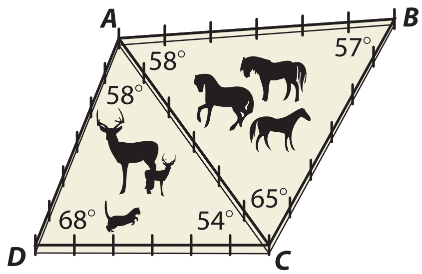


6. **Critique Reasoning** Dustin says that $\triangle FGH$ is an equilateral triangle because the sides appear to be the same length. Is his reasoning valid? Explain.

7. What additional information do you need to know before you can determine which side of the triangle is the longest? How can you find it?

8. Which side of the triangle is the longest? Explain how you found the answer.

The figure shows the angle measurements formed by two fenced-in animal pens that share a side. Use the figure for 9–10.



9. Caitlin says that $\overline{AC}$ is the longest segment of fencing because it is opposite 68°, the largest angle measure in the figure. Is her reasoning valid? Explain.

10. What is the longest segment of fencing in $\triangle ABC$? Explain your reasoning.

11. Find the longest segment of fencing in the figure. Explain your reasoning.

12. In triangle ABC, $\overline{AB}$ is longer than $\overline{BC}$ and $\overline{BC}$ is longer than $\overline{AC}$.

a. Draw a sketch of triangle ABC.

b. Name the smallest angle in the triangle. Explain your reasoning.

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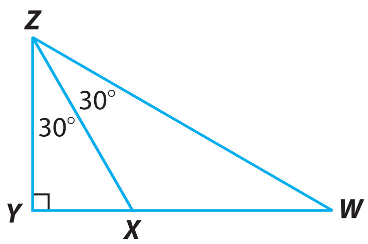
H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

13. **Persevere in Problem Solving** Determine the shortest line segment in the figure. Explain how you found the answer.



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14. **Communicate Mathematical Ideas** Explain how the relationship between the sides and angles of a triangle applies to isosceles triangles.

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15. **Critical Thinking** Can a scalene triangle contain a pair of congruent angles? Explain.

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